

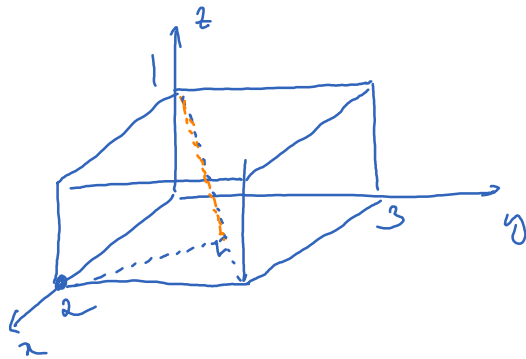
Lecture 4

Saturday, January 8, 2022 11:03 PM

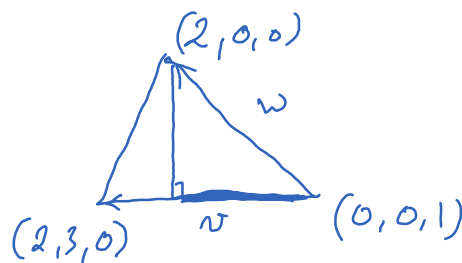
* Prayer

* Spiritual thought

Example on projection:

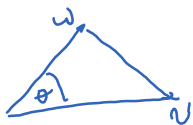


what is the length of the orange bar?



$$w = \langle 2, 0, -1 \rangle, \quad v = \langle 2, 3, -1 \rangle \implies \text{comp}_v w = \frac{v \cdot w}{|v|} = \frac{5}{\sqrt{14}}$$

Cross product: useful tool to compute area of a triangle or parallelogram.



$v \times w$ is a vector

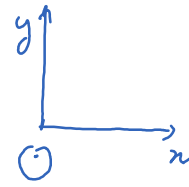
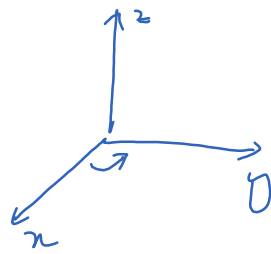
$v \cdot w$ is a number

$v \times w$ is a vector perpendicular to both v and w

$|v \times w| =$ area of parallelogram form by v and w

$v \times w$ is oriented by the right hand rule.

Right hand rule:



Algebraically,

$$v = \langle a, b, c \rangle$$

$$w = \langle d, e, f \rangle$$

$$v \times w = \left\langle \begin{vmatrix} b & c \\ e & f \end{vmatrix}, \begin{vmatrix} c & a \\ f & d \end{vmatrix}, \begin{vmatrix} a & b \\ d & e \end{vmatrix} \right\rangle$$

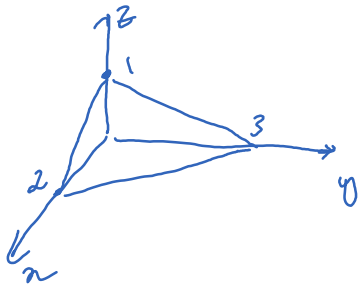
$$= \langle bf - ce, cd - af, ae - bd \rangle$$

Ex: $v = \langle 1, 2, 3 \rangle$

$$w = \langle 2, 1, -1 \rangle$$

$$v \times w = ?$$

Ex:

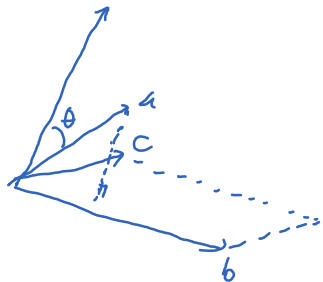


Find the area of the triangle

Ex: Check if three points are on the same line

Check if two vectors are parallel

$$\left. \begin{aligned} &\text{Triple product} \\ &a \cdot (b \times c) \\ &= \det \begin{pmatrix} a \\ b \\ c \end{pmatrix} \end{aligned} \right\}$$



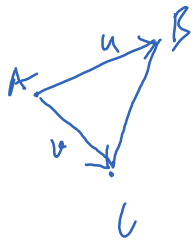
wait until
Wednesday

Ex Equation of a plane crossing 3 points:

$$A(1, 2, 3)$$

$$B(2, 3, 1)$$

$$C(-1, 1, 0)$$



$$u = \vec{AB} = \langle 1, 1, -2 \rangle$$

$$v = \vec{AC} = \langle -2, -1, -3 \rangle$$

$$\text{Normal vector: } u \times v = \langle -5, 7, 1 \rangle$$

$$M(x, y, z) \in \text{plane} \Leftrightarrow AM \perp (u \times v)$$

$$\Leftrightarrow \vec{AM} \cdot (u \times v) = 0$$

$$\Leftrightarrow \langle x-1, y-2, z-3 \rangle \cdot \langle -5, 7, 1 \rangle = 0$$

$$\Leftrightarrow -5(x-1) + 7(y-2) + (z-3) = 0$$

$$\Leftrightarrow \boxed{-5x + 7y + z = 12}$$